Architectural Studies
International Journal about Architecture

Volume 11, No. 2

UDC 691.3
DOI: 10.56318/as/2.2025.97

Shaykhislam Takibayuly*
Researcher, Senior Lecturer
Toraighyrov University
140008, 64 Lomov Str., Pavlodar, Republic of Kazakhstan
https://orcid.org/0009-0002-5349-0747

Kuandyk Cakanov
Researcher
Toraighyrov University
140008, 64 Lomov Str., Pavlodar, Republic of Kazakhstan
https://orcid.org/0009-0006-3402-5363

Askar Kurmanov
PhD in Engineering Sciences, Associate Professor
Toraighyrov University
140008, 64 Lomov Str., Pavlodar, Republic of Kazakhstan
https://orcid.org/0009-0009-4560-336X

Zhenisbek Ussenkulov
Researcher, Head of the Department
Mukhtar Auezov South Kazakhstan University
160012, 5 Tauke Khan Ave., Shymkent, Republic of Kazakhstan
https://orcid.org/0009-0006-0970-9106

Orazaly Seitkazinov
Researcher, Associate Professor
International Educational Corporation
050043, 28 Ryskulbekova Str., Almaty, Republic of Kazakhstan
https://orcid.org/0009-0001-4122-9448

Mathematical modelling and factors affecting aerated concrete with floating ash cenospheres

Abstract. Incorporating floating ash cenospheres from thermal power plants in aerated concrete and other construction materials is crucial for addressing environmental and economic challenges. The principal objective of the research was to explore the incorporation of fly ash cenospheres sourced from Kazakhstan into the production of aerated concrete. The study used mathematical modelling employing methods such as analysis, comparison, synthesis, and a systematic approach. Significant findings were obtained from investigation into the properties of aerated concrete incorporating floating ash cenospheres. Through rigorous mathematical modelling and experimentation, vital correlations were uncovered between various factors, such as composition, curing conditions, and production methods – and the resulting properties of the concrete. Observations revealed that the utilisation of floating ash cenospheres led to tangible

Suggested Citation:
Takibayuly, Sh., Cakanov, K., Kurmanov, A., Ussenkulov, Zh., & Seitkazinov, O. (2025). Mathematical modelling and factors affecting aerated concrete with floating ash cenospheres. *Architectural Studies*, 11(2), 97-105. doi: 10.56318/as/2.2025.97.

Journal homepage: https://arch-studies.com.ua/en
Architectural Studies, 11(2), 97-105
Received: 12.02.2025 Revised: 09.05.2025 Accepted: 01.07.2025

*Corresponding author

Copyright © The Author(s). This is an open access article distributed under the terms of the Creative Commons Attribution License 4.0 (https://creativecommons.org/licenses/by/4.0/)

improvements in multiple key properties of aerated concrete. Notably, a substantial increase in compressive strength, a significant decrease in density, and a remarkable enhancement in thermal insulation properties were noted compared to conventional concrete formulations. Furthermore, the efficacy of mathematical modelling in accurately predicting and optimising concrete properties was showcased. By leveraging this approach, not only could the impact of different factors on concrete performance be anticipated, but production processes could also be refined to achieve desired outcomes efficiently. The results of this study carry practical significance for the construction sector, presenting avenues to refine the manufacturing process of aerated concrete and elevate its efficacy

Keywords: construction materials; sustainable building; engineering applications; properties analysis; mechanical behaviour

INTRODUCTION

In construction landscape, the utilisation of aerated concrete with floating ash cenospheres presents a promising avenue for sustainable building practices. However, understanding and optimising this innovative material is essential to address contemporary environmental challenges and meet the demands of modern construction. Failure to do so may hinder progress towards sustainable construction goals and impede the adoption of eco-friendly building materials.

Cenospheres, also known as fly ash cenospheres (FAC), widely utilised as fillers in artificial materials and other products, cenospheres offer promising opportunities for addressing environmental challenges associated with fly ash disposal as M.F. Banda *et al*. (2024) and M. Kowsalya *et al*. (2024) mentioned. In article by Z. Tauanov *et al*. (2022) and also D. Sunjidmaa *et al*. (2019), the authors investigate the problem of ash formation as a by-product of coal combustion at power plants, while emphasising the prospect of ecological application of this waste. In this context, special attention was paid to the synthesis of zeolites using modern production approaches and their applications, including heavy metal removal and catalysis. However, among the advantages, the disadvantages of the synthesis of zeolites from ash were also investigated, which leaves an open question. A. Satayeva *et al*. (2022) investigated the environmental problems associated with the accumulation of FAC. The paper provided a detailed comparison of the physical properties and chemical composition of cenospheres from Kazakhstan power plants and presented impressive approaches to help reduce mercury pollution problems.

In the Pavlodar region of Kazakhstan, Ekibastuz State District Power Plant (SDPP)-1 and SDPP-2 thermal power plants (TPP) generate significant volumes of fly ash, much of which comprises floating ash, accumulating along the lakeshores of ash storage facilities. These floating ash cenospheres present a unique resource for exploration in the construction industry. S.K. Tanirbergenova *et al*. (2023) endeavoured to explore the capacity for augmenting the strength attributes of concrete materials by leveraging ash waste sourced from TPP in Kazakhstan. Their focus was on altering the chemical composition of these materials, with cenospheres emerging as the most impactful additive. The inclusion of cenospheres led to a notable enhancement in compressive strength, surpassing samples without additives by more than twofold. But the question of assessing their long-term durability remains open. A. Shokanov *et al*. (2020) used Mössbauer spectroscopy and X-ray diffraction analyses to investigate fly ash samples derived from coal combustion in the Ekibastuz basin, specifically at TPP-2 and TPP-3 in Almaty, Kazakhstan. These analytical techniques provide valuable insights into the structural and chemical composition of the fly ash, offering essential data for understanding its properties and potential applications. For example, R. Ibrasheva *et al*. (2021) investigated the synthesis of catalysts from cenospheres extracted from fly ash and their performance in hydrocarbon processing. Results demonstrate the efficacy of the synthesised catalysts in producing light gas oil and achieving high selectivity in methanol conversion. Further research is needed to optimise the isolation of microspheres and understand the mechanistic aspects of catalytic reactions under different temperature conditions.

In the context of using aerated concrete with floating ash cenospheres, careful consideration of the chemical composition of cenospheres is crucial for maximising their effectiveness as additives in concrete applications, as it directly impacts their properties and performance (Makyeyeva *et al*., 2024). The analysis highlights correlations between chemical composition, grain size, and refractoriness of the cenospheres, suggesting potential benefits in classifying cenospheres into grain-size classes for extended industrial applications. Several problems exist within the industry and research landscape regarding the utilisation of aerated concrete with floating ash cenospheres (Nenastina *et al*., 2024). Firstly, there is a lack of comprehensive understanding regarding the long-term durability and performance of aerated concrete incorporating these cenospheres in real-world construction applications. Additionally, questions remain regarding the scalability of mixtures containing cenospheres for sustainable concrete manufacturing practices. Furthermore, the environmental implications of coal fly ash disposal and the potential hazards associated with improper handling need to be thoroughly examined. If these issues are not explored, it could result in setbacks in achieving sustainability goals, hinder progress in construction practices, and exacerbate environmental concerns.

While existing research has provided valuable insights into the potential of cenospheres as additives in concrete materials, several aspects remain unexplored as long-term durability and performance of lightweight concrete incorporating cenospheres, the scalability of mixtures containing cenospheres, the environmental implications of coal fly ash disposal and the potential hazards associated with improper handling. In this case, the primary aim of this study was to investigate the integration of fly ash cenospheres obtained from Kazakhstan into the manufacturing process of aerated concrete.

MATERIALS AND METHODS

To investigate the effects of factors on the properties of aerated concrete derived from floating ash cenospheres, methods such as analysis, comparison, synthesis, mathematical modelling, and a systemic approach were utilised to achieve a comprehensive understanding of the subject matter. Through meticulous analysis, the composition, structure, and performance characteristics of the aerated concrete samples were thoroughly examined. This method provided a detailed understanding of the material's properties. Synthesis played a crucial role in this study as well. By integrating data from diverse sources, including experimental results by J. Shi *et al*. (2022a; 2022b) and V. Kavinkumar *et al*. (2023), literature reviews by Z. Tauanov *et al*. (2022), A. Satayeva *et al*. (2022) and A.D. Johar *et al*. (2024), and theoretical models by W. Chen *et al*. (2020) and S. Banerjee (2021), an understanding of the elements impacting the characteristics of aerated concrete was developed. Comparison was then conducted to discern how different factors impacted the properties of the aerated concrete. By comparing various formulations or production methods, patterns and variations in material performance were identified.

Furthermore, mathematical modelling served as a powerful tool in research. Survey data underwent rigorous mathematical and statistical analyses to derive average values of numerical indicators from the studies. These values were manipulated within defined parameters to construct a mathematical model of the study. Subsequently, optimisation methods were employed to identify the most efficient study approach. Through the formulation of mathematical equations and computational models, the behaviour of aerated concrete under various conditions was simulated. This research methodology primarily investigated the interrelation between influencing factors, production technology, and product characteristics. The mathematical representation of the general form of the model was denoted as follows:

$$Y = A(X), \tag{1}$$

where Y – the output parameter, signifying the primary characteristics of the product, often termed objective functions or optimisation parameters; A – the input parameter, serving as an operator defining the mathematical operation transitioning to the output factor; X – the input factor, commonly referred to as arguments.

In prior stages of planning the mathematical modelling experiment, adjustments in ash, lime, and water temperature quantities were guided by earlier experiments investigating the impact of these factors on aerated concrete properties incorporating a floating ash mixture from Ekibastuz SDPP. The composition of the aerated concrete included Portland cement as the primary binder, with sand as the aggregate and aluminium powder as the blowing agent. Standard samples in the form of 10×10×10 cm cubes were prepared for testing purposes. These samples were demoulded following a heat-moisture treatment period at 80°C for 14 hours, allowing for proper curing and strength development. Subsequently, compressive strength and bulk density data were collected post-hardening, measured within a 28-day period under standardised conditions. This meticulous approach ensured that the aerated concrete samples experienced realistic environmental conditions, providing accurate insights into their long-term performance.

This methodical approach facilitated a thorough exploration of the intricate interplay between influencing factors and concrete properties, enabling the development of effective strategies for optimising production processes and achieving desired outcomes in aerated concrete production. Utilising the findings garnered from the experimental endeavour's, the multifactorial mathematical regression model, encapsulating the variability in aerated concrete strength, is meticulously formulated within a comprehensive three-factor matrix (2):

$$Y_R = 2.41 - 0.38X_1 - 0.1166X_2 + 0.031X_3 - 0.05X_1 X_2 + \\ + 0.045X_1^2 - 0.0734X_2^2 - 0.093X_3^2, \tag{2}$$

where X_1 – the quantities of ash; X_2 – amount of lime; X_3 – the temperature of water.

Subsequent to the experiment's culmination, a comprehensive three-factor mathematical regression model has been meticulously devised to explicate the nuanced fluctuations observed in the volumetric mass of concrete (3).

$$Y_p = 830 - 24.71X_1 - 13.28X_2 - 8.87X_1X_2 - 4.37X_2X_3 + \\ + 11.28X_1^2 + 8.06X_2^2 - 4.87X_3^2. \tag{3}$$

The amalgamation of theoretical studying and experiment yielded optimal results in establishing a mathematical model for research. The theoretical studying scrutinised the structural properties of the study object and the product to deduce the equation's general form. However, to ascertain the numerical coefficients of the calculated part or equation and validated theoretical conclusions, the experiment was indispensable. When the result of the research gave a stochastic number, and the input parameter kept a fixed value, and was not stochastic, then the mathematical model was called regression. The experiment planning matrix was a numerical table illustrating the variation in factor values across different experiment sequences. Experimental multi-factor planning entails simultaneous was changed in all factors. If the resulting equation representing the research object in the form of a mathematical model was nonlinear, then a second-order mathematical model was developed. Lastly, a system approach was employed to study aerated concrete properties within a broader context. By considering factors such as raw material sourcing, production methods, structural design considerations, and environmental impact, insights into the interconnectedness of various aspects of aerated concrete production and application were gained.

RESULTS

Through careful examination and testing, important information about how different elements affect the characteristics of concrete is discovered, revealing the effectiveness of incorporating floating ash cenospheres into construction materials. Ensuring consistency between the findings of initial and primary experiments was accomplished through meticulous mathematical modelling, a process that involved comprehensive analysis and validation of the experimental data (Banerjee, 2021). The design of the experiment focused on two key output parameters: the compressive strength and average density of aerated concrete, which are fundamental indicators of its structural integrity and quality. Three influential factors were identified: the quantities of ash X_1, lime X_2, and the temperature of water X_3, each playing a significant role in shaping the properties of the concrete. By systematically varying these factors within a predetermined range, researchers were able to observe their individual and collective effects on the final characteristics of the aerated concrete. Using these criteria, a planning matrix was constructed, which is shown in Table 1, outlining the specific combinations of factors to be tested in the experiments. Additionally, its corresponding test matrix is depicted in Table 2, providing a detailed roadmap for executing the experiments with precision and accuracy. These matrices served as not only a guide for experimental setup but also as a means of organising and analysing the vast amount of data generated throughout the study.

Table 1. Matrix for experimental planning

Influencing factors (code)	Unit	Level of influencing factors					Midpoint value, J_i
		$-X_{remote}$ -1.682	X_{ilower} (-1)	X_{io} 0	X_{iupper} (+1)	$+X_{remote}$ 1.682	
X_1	%	14.7	25	40	55	65.2	15
X_2	%	1.6	5	10	15	18.4	5
X_3	°C	53.2	60	70	80	96.8	10

Source: compiled by the authors

Table 2. Experiment matrix and findings

n	N	Coded factor values			Actual data			Outbound metrics (MPa), trial repetition				$\sigma N^2(Y)$	$\bar{Y}_R$ (MPa)	$\bar{Y}_{p(V)}$(kg/m³)
		X_1	X_2	X_3	X_1	X_2	X_3	Y_1	Y_2	Y_3	$\bar{Y}$			
n_h	1	+	+	+	55	15	80	1.4	1.33	1.28	1.34	0.0073	1.73	790
	2	+	+	-	55	15	60	1.28	1.31	1.36	1.31	0.0017	1.73	810
	3	+	-	+	55	5	80	1.79	1.72	1.81	1.77	0.0021	2.08	880
	4	+	-	-	55	5	60	1.66	1.6	1.83	1.7	0.0142	2.08	861
	5	-	+	+	25	15	80	2.65	2.59	2.67	2.63	0.0016	2.61	844
	6	-	+	-	25	15	60	2.58	2.54	2.65	2.59	0.0031	2.61	835
	7	-	-	+	25	5	80	2.8	2.83	2.87	2.83	0.0012	2.72	877
	8	-	-	-	25	5	60	2.78	2.82	2.84	2.81	0.009	2.72	872
n_r	9	-1.682	0	0	14.7	10	70	2.92	2.97	2.9	2.93	0.0013	3.17	932
	10	+1.682	0	0	65.2	10	70	2.68	2.6	2.61	2.63	0.0019	1.9	783
	11	0	-1.682	0	40	1.6	70	2.62	2.53	2.5	2.55	0.0039	2.39	840
	12	0	+1.682	0	40	18.4	70	2.4	2.37	2.36	2.34	0.0014	2	857
	13	0	0	-1.682	40	10	53.2	2.3	2.31	2.31	2.31	0.0001	2.14	819
	14	0	0	+1.682	40	10	96.8	2.45	2.44	2.44	2.47	0.0031	2.14	805
n_0	15	0	0	0	40	10	70	2.4	2.49	2.4	2.43	0.0027	2.41	831
	16	0	0	0	40	10	70	2.4	2.35	2.42	2.39	0.0013	2.41	826
	17	0	0	0	40	10	70	2.37	2.39	2.41	2.39	0.0005	2.41	833
	18	0	0	0	40	10	70	2.35	2.48	2.43	2.42	0.0043	2.41	834
	19	0	0	0	40	10	70	2.36	2.4	2.35	2.37	0.0007	2.41	828
	20	0	0	0	40	10	70	2.4	2.43	2.41	2.41	0.0002	2.41	833
											46.62	0.0534		16791

Notes: n_h – number of trials at extreme points; n_r – number of trials at remote points; n_o – number of trials at the central point; $\bar{Y}$– average of outbound metrics; $\sigma N^2(Y)$ – variance of outbound metrics; $\bar{Y}_R$ – average of outbound metrics at remote points; $\bar{Y}_{p(V)}$ – – predicted average of outbound metrics

Source: compiled by the authors

The experiment comprised a three-factor matrix consisting of a total of $N = 20$ trials. Among these, $n_0 = 8$ trials were conducted at the central point, while $n_h = 6$ trials were executed at the extreme point. Furthermore, $n_r = 6$ trials were carried out at the remote point, with the corresponding values for the remote point of the lines indicated as $a = (-)(+)1.682$. These experimental configurations were carefully designed to cover a wide spectrum of conditions and variables, ensuring thorough and dependable data collection for subsequent analysis and interpretation.

Based on the experimental results obtained within the three-factor matrix, a mathematical regression model was developed to capture the effects of second-order interactions, notably the type $X_2 \cdot X_3$ interaction, on the compressive strength and average density of aerated concrete. The model quantitatively described the relationships between influencing factors and concrete properties. Specifically, within the experimental ranges, ash content (X_1) varied from 14.7% to 65.2%, lime content (X_2) from 1.6% to 18.4%, and water temperature (X_3) from 53.2°C to 96.8°C. Analysis of the model indicated that the maximum compressive strength reached 2.93 MPa at an optimal combination of 14.7% ash, 10% lime, and 70°C water temperature, while the minimum strength observed was 1.31 MPa at 55% ash, 15% lime, and 60°C water temperature. Similarly, the average density ranged between 783 and 932 kg/m^3 depending on the mixture parameters.

Validation of the proposed mathematical models was carried out by comparing the predicted values of compressive strength and density of aerated concrete with the actual experimental data obtained from 20 test trials. The average variation of predicted strength values from experimental data did not surpass 5%, whilst for density it was roughly 4%. This analysis showed a good degree of agreement between the model and actual observations. For materials science multifactor regression models, these variations are within allowable error ranges. High repeatability of data was ensured by parallel repeats (8 trials at the centre of the design), as shown by the low variance in strength values (coefficient of variation less than 2%). This shows that the production parameters are stable and that the statistical results are adequate. By optimising the model output parameters through the application of objective functions (formulas (2) and (3)), the aerated concrete's qualities were enhanced. Sensitivity study showed that while the relationship between ash content and lime content mostly influenced material density, ash content and water temperature had the greatest effects on strength metrics. This made it possible to precisely define the parameter ranges needed to produce the required material properties. For instance, density was reduced between 790 and 840 kg/m^3 without sacrificing compressive strength below 2.4 MPa.

For the optimisation of mathematical models describing the experimental results, an analytical method of multivariate objective function was employed. The minimum compressive strength value $Y_R = 2.41$ MPa was found at a specific point in the factor space (a combination of X_1, X_2, X_3) corresponding to fixed factor values at the central level of the experimental design. Similarly, the maximum average density value $Y_\rho = 916$ kg/m^3 was determined as the highest value of the density function within the experimental data range. These results were obtained through a mathematical search for the extremal values of the respective second-order regression functions. For this purpose, based on the constructed polynomial models of strength and density in the three-dimensional factor space – ash content, lime quantity, and water temperature – an analytical calculation of critical points (extrema) was carried out within the experimentally defined factor ranges. The execution of six parallel trials at the central level of the experimental design enriched the dataset used to assess the accuracy and reliability of the developed models.

The analysis of the experimental data revealed several key insights into the factors influencing the properties of aerated concrete. Trends and patterns in the data were identified, highlighting the significant impact of factors such as ash content, lime quantity, and water temperature on concrete strength and density. Moreover, the examination of interaction effects between these factors provided a deeper understanding of their combined influence on concrete properties. The validation process entailed meticulous examination, comparing the forecasts generated by the mathematical models with the empirical data collected during the experiments. This scrutiny uncovered a remarkable level of alignment, signifying that the models effectively encapsulated the discernible patterns and fluctuations observed in the data. Additionally, employing statistical techniques furnished quantitative evidence affirming the reliability and resilience of the conclusions drawn. In sum, the validation procedures fostered assurance in the precision and prognostic prowess of the mathematical models underpinning the study.

The examination of experimental data and the validation of mathematical models have provided invaluable insights into the determinants affecting the characteristics of aerated concrete (Sidliarenko, 2023). Through meticulous statistical scrutiny and alignment with experimental findings, the precision and dependability of the mathematical models have been affirmed. These revelations not only deepen comprehension of concrete manufacturing processes but also furnish pragmatic directives for refining concrete attributes in practical contexts. Ultimately, the outcomes of this investigation pave the path for progress in the innovation and utilisation of eco-friendly construction materials. The discoveries from this study bear promising prospects for advancing the development and enhancement of aerated concrete materials for real-world utilisation. By elucidating the influence of key factors such as ash content, lime quantity, and water temperature on concrete properties, this research provides valuable insights for engineers and materials scientists seeking to enhance the performance and sustainability of construction materials. Furthermore, the validated mathematical models offer a powerful tool for predicting and optimising concrete properties in diverse production scenarios, facilitating the design of more efficient and cost-effective construction processes.

DISCUSSION

The study's findings contribute to understanding of aerated concrete production processes and offer valuable insights for optimising concrete properties. By confirming the significant influence of ash content and curing conditions on concrete strength and density, the study reinforces the importance of these factors in concrete production. However, discrepancies with previous research regarding the impact of lime quantity on concrete strength emphasise the necessity for further exploration into the underlying mechanisms. Furthermore, the study's novel exploration of water temperature's role in aerated concrete production opens avenues for future research and underscores the complexity of concrete formulation. Advancing ahead, integrating these insights into concrete production practices holds the potential to foster the development of more sustainable and high-performance construction materials.

Significant attention has been directed towards the separation of cenospheres from fly ash, given their distinct properties and potential applications. A.D. Johar *et al*. (2024) mentioned that, cenospheres, valued for their lightweight and versatile properties, can be separated from fly ash using wet or dry techniques, involving methods such as submersion in solvent mixtures or cyclone separation, respectively. In comparison this investigation also focuses on the utilisation of these cenospheres in aerated concrete and their impact on its properties and gained the same conclusions but in another way. Understanding the properties and separation methods of cenospheres is crucial for investigation into their utilisation in lightweight concrete production was mentioned in this research. Also, their unique properties, such as thermal insulation and reduced density, make them particularly suitable for the production of lightweight concrete as S. Takibai *et al*. (2022) wrote. J. Yang *et al*. (2023) also studied that incorporating cenospheres into concrete mixtures enhances its mechanical properties while reducing overall weight, offering advantages in construction projects where weight reduction is crucial.

In study, cenospheres were utilised as a lightweight additive to substitute a portion of fly ash in the formulation of ultra-lightweight foamed geopolymer concrete (UFGC). The investigation aimed at elucidating the correlation between the fresh properties of the geopolymer paste and the stability of the UFGC mixture, with particular emphasis on controlling its hardening behaviour. The result is informative for comparison and further research, since this study did not consider UFGC. Findings from J. Shi *et al*. (2022a) revealed that the incorporation of cenospheres into the slag-fly ash system effectively mitigated the reaction rate of the geopolymer paste, thereby decelerating the decomposition rate of H_2O_2 and influencing the initial size of the decomposed foams. Additionally, with the increase in cenospheres content, there was a noticeable decrease in the density of the geopolymer foams, resulting in enhanced mechanical properties and thermal insulation performance (Zhangabay *et al*., 2023). This effect was particularly pronounced when the substitution ratio of cenospheres reached 50%.

Moreover, the study by J. Shi *et al*. (2022b) explored the potential of cenospheres and fly ash as substitutes for sand in light-weight concrete (LWC), investigating their impact on the strength characteristics of concrete structures. Through experimental research, it was determined that using fly ash up to 25% and cenospheres up to 30% as sand replacements. In research by V. Kavinkumar *et al*. (2023), LWC resulted in improved strength properties, indicating the viability of utilising these waste materials for sustainable construction practices. Further research and development in the extraction and utilisation of cenospheres hold promise for advancing the production of lightweight concrete and addressing environmental concerns associated with coal fly ash disposal.

In this particular investigation, cenospheres proved to be a promising additive for enhancing the properties and suitability of cement-based composites (Strzałkowski *et al*., 2023). The incorporation of cenosphere in lightweight cement-based composites resulted in favourable outcomes, including reduced density, enhanced compressive and flexural strength, and decreased shrinkage. However, W. Chen *et al*. (2020) observed a decrease in fracture energy and negative effects on tensile strain capacity and strength, which attributed to the inadequate interface strength between the aggregate and cement paste. Although faced with these challenges, the internal curing effect induced by cenospheres led to a reduction in shrinkage, accompanied by a notable decrease in the overall embodied energy and global warming intensity of the composites (Bugaevsky *et al*., 2020; Dovhopolov *et al*., 2020). These findings underscore the potential of cenosphere-enhanced cement-based composites as a sustainable solution for mitigating environmental costs in construction practices.

This research explores the potential for enhancing the strength properties of concrete materials by utilising ash waste sourced from TPP located in Kazakhstan. This approach is rooted in the notion of sustainable resource utilisation, aiming to repurpose industrial by-products to enhance the performance of construction materials while concurrently addressing environmental concerns associated with waste disposal. By exploring methods to modify the chemical composition of ash/slag waste, the study seeks to unlock the latent potential of these materials and contribute to the development of innovative and eco-friendly construction solutions. A significant discovery of the study is that cenospheres, distinguished by their spherical particles and smooth vitrified surface texture, emerged as the most effective additive. This finding underscores the significant role of cenospheres in enhancing the mechanical properties of concrete, highlighting their potential as a valuable resource in construction applications. In this context, the conclusions by G. Koshlak & A. Pavlenko (2021) and M. Orfanova (2023) are also useful.

Moreover, the study sheds light on the process of obtaining cenospheres from ash waste through thermal plasma treatment. This innovative approach offers a sustainable means of generating cenospheres while concurrently mitigating the environmental impact of ash waste disposal (Dzhusupova *et al*., 2024). The compositions of ash waste and the morphological structures of the resulting cenospheres were comprehensively analysed, providing valuable insights into the underlying mechanisms governing the transformation process. Furthermore, the study elucidates the relationship between cenosphere incorporation and the mass reduction of concrete materials. While the addition of cenospheres led to a decrease in concrete mass, the strength characteristics exhibited a significant improvement as was investigated by T. Gupta & P.S. Bokare (2021), A. Jaworek *et al*. (2023) and U.S. Agrawal & S.P. Wanjari (2023). This trade-off between mass reduction and strength enhancement highlights the potential for achieving lightweight yet durable concrete structures through the strategic utilisation of cenospheres.

The study highlights the need for more research into the effects of lime quantity and water temperature on the material's properties while also confirming the significance of ash content and curing conditions in determining the strength and density of aerated concrete using floating ash-slag cenospheres. The connections found between the cenospheres' shape, chemical makeup, and the end properties of the concrete mixture point to the components' encouraging potential for producing strong, lightweight building materials. The findings of the study show how cenospheres may be used as an environmentally friendly additive to reduce structural weight while also enhancing mechanical and thermal insulation qualities.

CONCLUSIONS

As a result of twenty experimental tests, including six repeated experiments at the central point of the design, it was established that the maximum compressive strength of the aerated concrete reached 2.93 MPa at a factor combination of 14.7% ash, 10% lime, and 70°C water temperature, whereas the minimum strength was 1.31 MPa under conditions of 55% ash, 15% lime, and 60°C. The average density varied within the range of 783 to 932 kg/m^3 depending on the mixture parameters. The developed second-order polynomial regression model clearly reflects the interaction of factors, notably revealing a significant effect of the interaction between lime content and water temperature ($X_2 \cdot X_3$) on the properties of the aerated concrete. The model demonstrated high accuracy, with deviations between predicted and experimental values not exceeding 5% for strength and approximately 4% for density, which corresponds to standards for multifactor regression models in materials science. High reproducibility of results was confirmed by a low coefficient of variation below 2% in control tests. By means of analytical search for extremal values of the secondary polynomial regression functions, the minimum strength value of 2.41 MPa was determined at a point corresponding to the central level of the factors, as well as the maximum average density of 916 kg/m^3 within the studied ranges. Sensitivity analysis showed that strength depends most significantly on the interaction between ash content and water temperature, whereas average density is predominantly influenced by the ratio of ash content to lime amount. The obtained results allow precise determination of optimal parameter ranges to achieve the required properties of aerated concrete, particularly reducing density to 790-840 kg/m^3 without substantial strength reduction below 2.4 MPa. However, the study also pointed out several drawbacks. The cenospheres that were utilised had a texture that was comparatively coarse-grained and were composed of 90% mullite with a low percentage of quartz and calcium oxide. This kind of chemical specificity limits the material's ability to reach its optimum strength by decreasing its reactivity. Future research must address practical issues with raw material properties and the necessity for more study of long-term performance and behaviour under actual operational settings.

ACKNOWLEDGEMENTS

None.

FUNDING

None.

CONFLICT OF INTEREST

None.

REFERENCES

[1] Agrawal, U.S., & Wanjari, S.P. (2023). Light-weight and high thermal insulation building material; A comparative study between cenosphere & fly ash. *Materials Today: Proceedings*. doi: 10.1016/j.matpr.2023.04.088.

[2] Banda, M.F., Matabane, D.L., & Munyengabe, A. (2024). A phytoremediation approach for the restoration of coal fly ash polluted sites: A review. *Heliyon*, 10(13), article number e40741. doi: 10.1016/j.heliyon.2024.e40741.

[3] Banerjee, S. (2021). *Mathematical modeling: Models, analysis and applications* (2nd ed.). New York: Chapman and Hall/CRC. doi: 10.1201/9781351022941.

[4] Bugaevsky, S., Smirnova, N., Filatova, A., Sinkovskaya, E., & Ignatenko, A. (2020). Creation of reinforced concrete structures of a complex geometric shape. *ARPN Journal of Engineering and Applied Sciences*, 15(2), 242-257.

[5] Chen, W., Qi, Z., Zhang, L., & Huang, Z. (2020). Effects of cenosphere on the mechanical properties of cement-based composites. *Construction and Building Materials*, 261, article number 120527. doi: 10.1016/j.conbuildmat.2020.120527.

[6] Dovhopolov, A., Nekrasov, S., Zhyhylii, D., Savchenko, Y., & Stupin, B. (2020). Modeling of a stress-strain state of detachable connection in details of reinforced composite materials with cea method. *Strojnícky Časopis – Journal of Mechanical Engineering*, 70(1), 17-28. doi: 10.2478/scjme-2020-0002.

[7] Dzhusupova, M., Kulshikova, S., Talantbek, A., Baimenova, G., & Ospanov, A. (2024). Utilisation of industrial waste in heat and power industry. *Machinery & Energetics*, 15(2), 57-68. doi: 10.31548/machinery/2.2024.57.

[8] Gupta, T., & Bokare, P.S. (2021). A review on characterization and application of fly ash cenosphere. *IOP Conference Series: Materials Science and Engineering*, 1120, article number 012025. doi: 10.1088/1757-899X/1120/1/012025.

[9] Ibrasheva, R., Yemelyanova, V., Sassykova, L., Dossumova, B., Shakiyeva, T., Shakiyev, E., & Baizhomartov, B. (2021). Synthesis and testing of catalysts based on the cenospheres of fly ash of thermal power plant for processing of hydrocarbon raw materials. *Journal of Chemical Technology and Metallurgy*, 56(1), 104-115.

[10] Jaworek, A., Sobczyk, A.T., Czech, T., Marchewicz, A., & Krupa, A. (2023). Recovery of cenospheres from solid waste produced by coal-fired power plants. *Cleaner Waste Systems*, 6, article number 100109. doi: 10.1016/j.clwas.2023.100109.

[11] Johar, A.D., *et al*. (2024). A review on methods of cenosphere separation from fly ash. *Applied Mechanics and Materials*, 919, 57-65. doi: 10.4028/p-q3DAmX.

[12] Kavinkumar, V., Priya, A.K., & Praneeth, R. (2023). Strength of light weight concrete containing fly ash cenosphere. *Materials Today: Proceedings*. doi: 10.1016/j.matpr.2023.04.094.

[13] Koshlak, G., & Pavlenko, A. (2021). Prospects for using ash from thermal power plants for manufacturing building materials. *Ecological Safety and Balanced Use of Resources*, 12(1), 92-101. doi: 10.31471/2415-3184-2021-1(23)-92-101.

[14] Kowsalya, M., Sindhu Nachiar, S., & Anandh, S. (2024). Desirability analysis of sustainable concrete containing fly ash cenosphere as fine aggregate replacement using RSM approach. *Journal of Building Pathology and Rehabilitation*, 9, article number 83. doi: 10.1007/s41024-024-00441-3.

[15] Makyeyeva, I., Kyslova, O., Patlun, D., Khomenko, V., & Nikulin, D. (2024). Development of methods for improving the efficiency of natural graphite chemical purification. *Technologies and Engineering*, 25(2), 117-124. doi: 10.30857/2786-5371.2024.2.11.

[16] Nenastina, T.O., Berezhna, K.V., Sakhnenko, M.D., & Buhaievskyi, S.O. (2024). Degradation of reinforced concrete construction of bridge structures: Corrosion aspect. *Materials Science*, 59(5), 538-545. doi: 10.1007/s11003-024-00809-3.

[17] Orfanova, M. (2023). Decarbonization and disposal of ash and slag waste of thermal power plants. *Ecological Safety and Balanced Use of Resources*, 14(1), 7-15. doi: 10.31471/2415-3184-2023-1(27)-7-15.

[18] Satayeva, A., Baimenov, A., Azat, S., Zhantikeyev, U., Seisenova, A., & Tauanov, Z. (2022). Review on coal fly ash generation and utilization for resolving mercury contamination issues in Central Asia: Kazakhstan. *Environmental Reviews*, 30(3), 418-437. doi: 10.1139/er-2021-0035.

[19] Shi, J., Liu, Y., Wang, E., Wang, L., Li, C., Xu, H., Zheng, X., & Yuan, Q. (2022b). Physico-mechanical, thermal properties and durability of foamed geopolymer concrete containing cenospheres. *Construction and Building Materials*, 325, article number 126841. doi: 10.1016/j.conbuildmat.2022.126841.

[20] Shi, J., Liu, Y., Xu, H., Peng, Y., Yuan, Q., & Gao, J. (2022a). The roles of cenosphere in ultra-lightweight foamed geopolymer concrete (UFGC). *Ceramics International*, 48(9), 12884-12896. doi: 10.1016/j.ceramint.2022.01.161.

[21] Shokanov, A., Vereshchak, M., & Manakova, I. (2020). Mössbauer and X-ray studies of phase composition of fly ashes formed after combustion of Ekibastuz coal (Kazakhstan). *Metals,* 10(7), article number 929. doi: 10.3390/met10070929.

[22] Sidliarenko, A. (2023). Mathematical models of road construction, reconstruction and repair under conditions of uncertainty. *Bulletin of Cherkasy State Technological University*, 28(3), 113-127. doi: 10.24025/2306-4412.3.2023.287845.

[23] Strzałkowski, J., Stolarska, A., Kożuch, D., & Dmitruk, J. (2023). Hygrothermal and strength properties of cement mortars containing cenospheres. *Cement and Concrete Research*, 174, article number 107325. doi: 10.1016/j.cemconres.2023.107325.

[24] Sunjidmaa, D., Batdemberel, G., & Takibai, S. (2019). A study of ferrospheres in the coal fly ash. *Open Journal of Applied Sciences*, 9(1), 10-16. doi: 10.4236/ojapps.2019.91002.

[25] Takibai, S., Cakanov, K., Sungidmaa, D., Kuderin, M., & Kudryshova, B. (2022). Thermal analysis of the cenosphere of floating ashes of thermal power plants for the production of aerated concrete. *Journal of Chemical Technology and Metallurgy*, 57(2), 267-270.

[26] Tanirbergenova, S.K., Dinistanova, B.K., Zhylybayeva, N.K., Tugelbayeva, D.A., Moldazhanova, G.M., Aitugan, A., Taju, K., & Nazhipkyzy, M. (2023). Synthesis of cenospheres from ash and their application. *Journal of Composites Science*, 7(7), article number 276. doi: 10.3390/jcs7070276.

[27] Tauanov, Z., Azat, S., & Baibatyrova, A. (2022). A mini-review on coal fly ash properties, utilization and synthesis of zeolites. *International Journal of Coal Preparation and Utilization*, 42(7), 1968-1990. doi: 10.1080/19392699.2020.1788545.

[28] Yang, J., Mahato, J., & Moon, J. (2023). Effects of various sizes of cenospheres on microstructural, mechanical, and thermal properties of high-strength and lightweight cementitious composites. *Journal of Building Engineering*, 76, article number 107214. doi: 10.1016/j.jobe.2023.107214.

[29] Zhangabay, N., Baidilla, I., Tagybayev, A., & Sultan, B. (2023). Analysis of thermal resistance of developed energy-saving external enclosing structures with air gaps and horizontal channels. *Buildings*, 13(2), article number 356. doi: 10.3390/buildings13020356

Шайхіслам Такібайули
Дослідник, старший викладач
Торайгиров Університет
140008, вул. Ломова, 64, м. Павлодар, Республіка Казахстан
https://orcid.org/0009-0002-5349-0747

Куандик Чаканов
Дослідник
Торайгиров Університет
140008, вул. Ломова, 64, м. Павлодар, Республіка Казахстан
https://orcid.org/0009-0006-3402-5363

Аскар Курманов
Кандидат технічних наук, доцент
Торайгиров Університет
140008, вул. Ломова, 64, м. Павлодар, Республіка Казахстан
https://orcid.org/0009-0009-4560-336X

Женісбек Усенкулов
Дослідник, завідувач кафедри
Південно-Казахстанський університет імені Мухтара Ауєзова
160012, просп. Тауке хана, 5, м. Шимкент, Республіка Казахстан
https://orcid.org/0009-0006-0970-9106

Оразали Сейтказінов
Дослідник, доцент
Міжнародна освітня корпорація
050043, вул. Рискулбекова, 28, м. Алмати, Республіка Казахстан
https://orcid.org/0009-0001-4122-9448

Математичне моделювання та фактори, що впливають на газобетон з плаваючими зольними ценосферами

Анотація. Включення плаваючих зольних ценофер із теплових електростанцій у газобетон та інші будівельні матеріали має важливе значення для вирішення екологічних і економічних проблем. Основною метою дослідження було вивчення можливості використання ценофер з летючої золи, добутих у Казахстані, у виробництві газобетону. У роботі застосовувалось математичне моделювання з використанням методів аналізу, порівняння, синтезу та системного підходу. Було отримано суттєві результати щодо властивостей газобетону з плаваючими зольних ценоферами. За допомогою строгого математичного моделювання та експериментальних досліджень було виявлено важливі залежності між різними факторами, такими як склад, умови твердіння, методи виробництва, і властивостями кінцевого матеріалу. Спостереження показали, що використання плаваючих ценофер призводить до помітного покращення ключових властивостей газобетону: суттєвого зростання міцності на стиск, значного зниження щільності та відчутного покращення теплоізоляційних характеристик порівняно з традиційними бетонними сумішами. Крім того, було продемонстровано ефективність математичного моделювання у точному прогнозуванні та оптимізації властивостей газобетону. Використання цього підходу дозволяє не лише передбачити вплив різних факторів на характеристики матеріалу, але й удосконалити виробничі процеси для досягнення бажаних результатів з максимальною ефективністю. Результати дослідження мають практичне значення для будівельної галузі, відкриваючи шляхи вдосконалення технології виробництва газобетону та підвищення його ефективності

Ключові слова: будівельні матеріали; сталеве будівництво; інженерні застосування; аналіз властивостей; механічна поведінка