Architectural Studies
International Journal
about Architecture

Volume 9, No. 2

UDC 624.012.45
DOI: 10.56318/as/2.2023.35

Mukhlis Hajiyev
Doctor of Technical Sciences, Professor
Azerbaijan University of Architecture and Construction
AZ1073, 11 Ayna Sultanova Str., Baku, Azerbaijan
https://orcid.org/0000-0001-6782-0941

Masil Damirov*
Postgraduate Student, Senior Lecturer
Azerbaijan University of Architecture and Construction
AZ1073, 11 Ayna Sultanova Str., Baku, Azerbaijan
https://orcid.org/0000-0002-3779-4305

Stress-strain state and bearing capacity of compressed reinforced concrete elements of annular section

Abstract. The study of the stress-strain state and bearing capacity of compressed reinforced concrete elements of the annular section is of urgent importance, considering the ever-growing need to improve and optimise infrastructure facilities, such as bridges and supports, to ensure their reliability and safety. The purpose of this study is to investigate and analyse the stress-strain state of compressed reinforced concrete elements of annular section in order to determine their bearing capacity and improve the efficiency of design and construction of infrastructure facilities. The analytical method, classification, functional, statistical, synthesis, and other methods used in the paper should be highlighted. Compressed reinforced concrete elements of annular cross-section are widely used in various fields of engineering and construction. However, since the 90s of the 20th century, there has been a noticeable development of nonlinear deformation theory for the calculation of reinforced concrete structures, which complicates the analysis due to the specific features of round and annular sections and leads to the absence of simple analytical methods. This study presents an effective numerical technique for investigating the stress-strain state and bearing capacity of non-centrally compressed elements, using a linear fractional dependence for concrete under compression, approved in the Eurocode, and a symmetrical two-line diagram of reinforcement deformation. It is important to emphasise that this methodology avoids the need to classify elements as short or long, and to separately account for small and large eccentricities, as these aspects are automatically considered in the calculations. Confirmation of the effectiveness of this technique was obtained through the results of numerical experiments. The practical significance of this research lies in the development of more accurate and reliable methods for calculating and designing compressed reinforced concrete elements of annular section, which contributes to increasing the safety and durability of infrastructure facilities and reducing the risk of destruction

Keywords: reinforcement; eccentricity; load-deflection diagram; infrastructure facilities; numerical technique

---

Suggested Citation:
Hajiyev, M., & Damirov, M. (2023). Stress-strain state and bearing capacity of compressed reinforced concrete elements of annular section. *Architectural Studies*, 9(2), 35-46. doi: 10.56318/as/2.2023.35.

Journal homepage: https://arch-studies.com.ua/en#
Architectural Studies, 9(2), 35-46
Received: 17.08.2023 Revised: 11.10.2023 Accepted: 15.12.2023

*Corresponding author

INTRODUCTION

The research topic is defined is becoming important in light of the ever-growing need for modernization and optimization of infrastructure facilities, such as bridges and supports. These elements are an integral part of many engineering structures, and their reliability, safety, and efficiency are key aspects to ensure the sustainability of the infrastructure. Given the complex geometry of annular sections and current trends in nonlinear deformation theory, the development of accurate and reliable calculation methods becomes an urgent task, since it minimises the risks of destruction and reduces construction and maintenance costs. Thus, the study of this topic has a direct practical impact on the safety and efficiency of infrastructure facilities and buildings, which makes it relevant and important for the scientific and engineering community.

The problem of the study is that the compressed reinforced concrete elements of the annular section are complex structures with an inhomogeneous distribution of stresses and deformations, and as of 2023, there are no generally accepted analytical methods capable of providing an accurate calculation of their bearing capacity. The features of circular and annular sections make analysis more difficult, and existing traditional calculation methods often cannot be applied without additional simplifications, which can lead to inaccurate results and underestimation of the safety of the structure. Therefore, it is necessary to develop more efficient and accurate numerical methods that can consider all the features of such elements and provide a reliable analysis of their stress-strain state, which is of key importance for ensuring the safety and reliability of infrastructure facilities.

Researchers A. Janahmadov *et al.* (2021) noted that with the increasing complexity of engineering structures, such as bridges and supports, there is a need for more accurate and reliable calculation methods not only to ensure safety, but also to reduce construction and maintenance costs. Effective engineering solutions based on accurate calculations can contribute to the long-term reliability of structures and reduce operating costs, which is important for the sustainability of infrastructure, buildings and the economic efficiency of projects.

In a study conducted by N. Baghirzade (2022), it is noted that the calculation of wooden elements based on the model of nonlinear deformation is an important engineering tool for determining the behaviour of wooden structures under load. This technique allows considering nonlinear deformations, which significantly increases the accuracy of calculations and the safety of construction objects made of wood.

Kh. Seyfullaev & A. Garaev (2018) suggest using linear fractional dependence for concrete under compression, which may be an important element in the development of more efficient numerical methods for reinforced concrete elements. This dependence can become a key component in the development of more efficient numerical methods that can more accurately assess the stress-strain state and load-bearing capacity of such structures. It allows a more accurate account of the concrete's behaviour in compression, which helps to improve the accuracy of the calculations.

T. Azizov (2021) emphasises the complexity of classifying elements as short or long, and the need to consider small and large eccentricities in the analysis of compressed elements, which requires more universal methods. The analysis of compressed elements of the annular section becomes more complicated due to the classification of elements as short or long and taking into account various eccentricities. This requires the development of universal analysis methods that consider a variety of parameters and conditions for more accurate design and calculations.

The study by B. Jian *et al.* (2023) confirms that the calculation of a compressed reinforced concrete element of circular cross-section according to a three-line concrete compression scheme is an important stage in the design of reinforced concrete structures. This technique considers the nonlinear behaviour of concrete during compression, which allows more accurately determining the bearing capacity of the element and ensuring its safety under various loads.

The study aimed to analyse the stress-strain condition of compressed reinforced concrete elements with an annular section, including assessing bearing capacity, identifying stress-strain parameters under various loads, and enhancing engineering structure design and construction procedures.

MATERIALS AND METHODS

The analytical method helped in the formalisation of the basic principles and patterns underlying the behaviour of compressed reinforced concrete elements of the annular section. This method established a connection between the geometric, material and load parameters of structures, which became the basis for the development of more complex and effective methods of calculation and design. The analytical method also allowed conducting theoretical studies and determining the influence of various factors on the stress-strain state of the elements, which made it possible to develop more accurate and reliable methods for predicting the bearing capacity and behaviour of structures under various operating conditions.

Using the functional method, models were created focused on describing the behaviour of compressed reinforced concrete elements of the annular section depending on various functional influences and conditions. This method allowed developing functional dependencies and equations describing the relationship between impacts and responses of structures. The application of the functional method provided a tool for a more detailed study and optimisation of projects, allowing the authors to consider various scenarios and impacts on compressed elements. This contributed to a more flexible approach to design and helped to better understand the variability of conditions, which ultimately increased the reliability and efficiency of engineering structures of the annular section.

The structural and functional method helped in a deeper understanding of the internal organisation of the compressed reinforced concrete elements of the annular section and their functional characteristics. This method revealed the relationship between the structure of facilities and their function in the context of various loads and operating conditions. The application of the structural and functional method has enriched the knowledge of how changes in geometry, material and reinforcement affect the mechanical characteristics of elements. This allowed developing more optimal structures adapted to the conditions of the task, contributing to an increase in their bearing capacity and reliability.

The deduction method helped in the study of the initial principles and logical patterns underlying the behaviour of compressed reinforced concrete elements of the annular section. The application of this method allowed the researchers to identify the basic principles on which the constructions are based, and logically deduce patterns and dependencies between different parameters. This provided a deeper understanding of the processes occurring inside the elements, and allowed developing more accurate theoretical models to describe their behaviour. The deduction method became the basis for the development of more accurate and predictable methods of calculation and design, which, in turn, contributed to improving the reliability and safety of compressed reinforced concrete elements.

ABC Algorithmic Language was used in this research to perform mathematical calculations. This approach allowed to manipulate variables, observe outcomes, and draw conclusions about the language's effectiveness in handling specific formulas.in combination with quantitative analyse, it allowed to work with numerical data generated through the implementation of mathematical formulas in the ABC Algorithmic Language.

RESULTS

The investigation of load-bearing capacity is an essential component of the study aimed at determining the maximum load that compressed reinforced concrete elements of the annular section can withstand until possible destruction or violation of rigidity conditions (Sadeghian *et al.*, 2021). This stage of research is of critical importance in the design and construction process, as it provides an opportunity for engineers to guarantee the safety and reliability of the structure in real operational conditions. The study of the stress-strain state and bearing capacity of compressed reinforced concrete elements of the annular section includes the analysis of the mechanical properties of materials, analytical and numerical calculations, physical experiments, and the creation of mathematical models (Zhang *et al.*, 2022). These efforts are aimed at a deeper understanding and improvement of the design and construction of such elements. In fact, this research plays a key role in ensuring the safety and efficiency of engineering structures where compressed reinforced concrete elements of the annular section are used. Diagram of concrete deformation under compression in the form proposed by Eurocode (1) (Spirande *et al.*, 2023):

$$\sigma_b = R_b \cdot \frac{k \cdot \frac{\varepsilon_b}{\varepsilon_R} - \left(\frac{\varepsilon_b}{\varepsilon_R}\right)^2}{1 + (k - 2) \cdot \frac{\varepsilon_b}{\varepsilon_R}}, \tag{1}$$

where ε_R – deformation corresponding to the maximum of the stress-strain diagram; ε_b – initial modulus of concrete deformation.

When developing the calculation methodology, it is assumed that concrete does not work for tension, stresses in the cross-section are completely accepted by reinforcement, for a complex cross-section up to the exhaustion of the bearing capacity, the hypothesis of flat sections is valid (Ruiz *et al.*, 2023). Depending on the flexibility and eccentricity of the compressive force, four options for the location of the neutral axis and the distribution of compressive stresses in concrete along the cross section are possible, as shown in Figure 1.

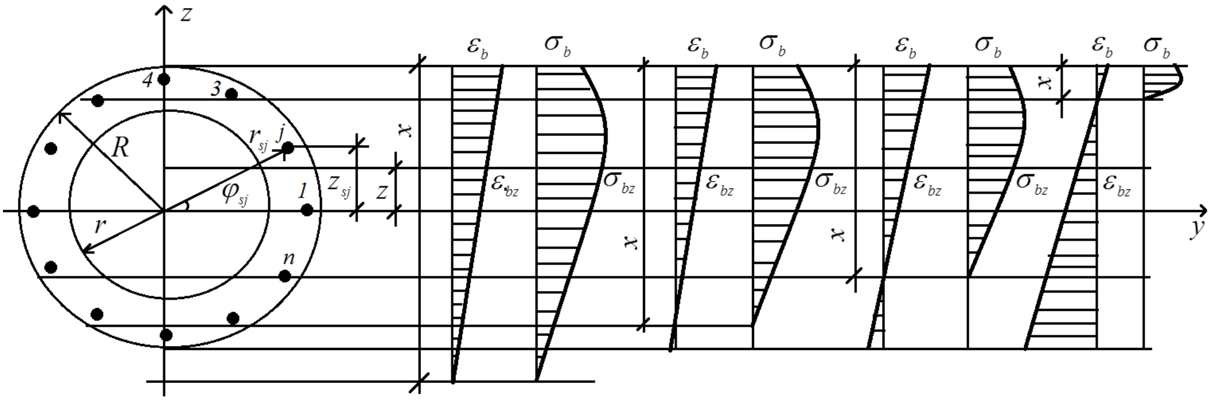

Figure 1. Design scheme of the annular section

Source: compiled by the authors based on G. Ruiz *et al.* (2023)

Based on the assumption of flat sections, with respect to the distribution of deformations across the section, the following statement can be formulated: $\varepsilon_{bz} = \frac{\varepsilon_b}{x} \cdot (x - R + z)$. After substituting this expression in (1) and introducing the notation $\beta = \frac{\varepsilon_b}{\varepsilon_R}$ – level of deformations of the compressed section face, $\xi = \frac{x}{R}$ –

dimensionless parameter of the neutral axis, $\bar{z} = \frac{z}{R}$ – dimensionless coordinate, for the distribution of compressive stresses in concrete, it is possible to obtain (2):

$$\sigma_{bz} = R_b \cdot \frac{k \cdot \frac{\beta}{\xi} \cdot (\xi - 1 + \bar{z}) - \left(\frac{\beta}{\xi}\right)^2 \cdot (\xi - 1 + \bar{z})^2}{1 + (k - 2) \cdot \frac{\beta}{\xi} \cdot (\xi - 1 + \bar{z})}. \tag{2}$$

Using equation (2) to determine the normal force and bending moment caused by compressive stresses in concrete, provided that the neutral axis is outside the section (that is, at specific parameter values), the following can be derived (3-5):

$$N_{bR} = 2 \cdot R^2 \cdot R_b \cdot N^*_{bR}(\beta, \xi); \; M_{bR} = 2 \cdot R^3 \cdot R_b \cdot M^*_{bR}(\beta, \xi), \tag{3}$$

$$N^*_{bR}(\beta, \xi) = \int_{-1}^{1} \frac{k \cdot \frac{\beta}{\xi} \cdot (\xi - 1 + \bar{z}) - \left(\frac{\beta}{\xi}\right)^2 \cdot (\xi - 1 + \bar{z})^2}{1 + (k - 2) \cdot \frac{\beta}{\xi} \cdot (\xi - 1 + \bar{z})} \cdot \sqrt{1 - \bar{z}^2} \cdot d\bar{z}, \tag{4}$$

$$M^*_{bR}(\beta, \xi) = \int_{-1}^{1} \frac{k \cdot \frac{\beta}{\xi} \cdot (\xi - 1 + \bar{z}) - \left(\frac{\beta}{\xi}\right)^2 \cdot (\xi - 1 + \bar{z})^2}{1 + (k - 2) \cdot \frac{\beta}{\xi} \cdot (\xi - 1 + \bar{z})} \cdot \bar{z} \cdot \sqrt{1 - \bar{z}^2} \cdot d\bar{z}. \tag{5}$$

Similarly, when the neutral axis passes outside the section of the inner circle and inside the outer circle, i.e., at $\xi \geqslant 1 + \gamma$, where $\gamma = r/R$ is the ratio of the radii of the inner and outer circles, to determine the normal force and bending moment caused by compressive stresses in concrete, the following results can be obtained (6-8):

$$N_{br} = 2 \cdot R^2 \cdot R_b \cdot N^*_{br}(\beta, \xi); \; M_{br} = 2 \cdot R^3 \cdot R_b \cdot M^*_{br}(\beta, \xi), \tag{6}$$

$$N^*_{br}(\beta, \xi) = \int_{-\gamma}^{\gamma} \frac{k \cdot \frac{\beta}{\xi} \cdot (\xi - 1 + \bar{z}) - \left(\frac{\beta}{\xi}\right)^2 \cdot (\xi - 1 + \bar{z})^2}{1 + (k - 2) \cdot \frac{\beta}{\xi} \cdot (\xi - 1 + \bar{z})} \cdot \sqrt{\gamma^2 - \bar{z}^2} \cdot d\bar{z}, \tag{7}$$

$$M^*_{br}(\beta, \xi) = \int_{-\gamma}^{\gamma} \frac{k \cdot \frac{\beta}{\xi} \cdot (\xi - 1 + \bar{z}) - \left(\frac{\beta}{\xi}\right)^2 \cdot (\xi - 1 + \bar{z})^2}{1 + (k - 2) \cdot \frac{\beta}{\xi} \cdot (\xi - 1 + \bar{z})} \cdot \bar{z} \cdot \sqrt{\gamma^2 - \bar{z}^2} \cdot d\bar{z}. \tag{8}$$

Based on the expressions obtained for this case, when the neutral axis passes outside the section of the inner circles, it is possible to find (9, 10):

$$N_b = N_{bR} - N_{br} = 2 \cdot R^2 \cdot R_b \cdot \left(N^*_{bR}(\beta, \xi) - N^*_{br}(\beta, \xi)\right), \tag{9}$$

$$M_b = M_{bR} - M_{br} = 2 \cdot R^3 \cdot R_b \cdot \left(M^*_{bR}(\beta, \xi) - M^*_{br}(\beta, \xi)\right). \tag{10}$$

The following two functions are introduced (11, 12):

$$Y_{Nb}(\beta, \xi, a) = \int_a^b f(\beta, \xi, \bar{z}) \cdot \sqrt{a^2 - \bar{z}^2} \cdot d\bar{z}, \tag{11}$$

$$Y_{Mb}(\beta, \xi, a, b) = \int_a^b f(\beta, \xi, \bar{z}) \cdot z \cdot \sqrt{a^2 - \bar{z}^2} \cdot d\bar{z}. \tag{12}$$

The designation (13) is introduced here:

$$f(\beta, \xi, \bar{z}) = \frac{k \cdot \frac{\beta}{\xi} \cdot (\xi - 1 + \bar{z}) - \left(\frac{\beta}{\xi}\right)^2 \cdot (\xi - 1 + \bar{z})^2}{1 + (k - 2) \cdot \frac{\beta}{\xi} \cdot (\xi - 1 + \bar{z})}. \tag{13}$$

Hence, depending on the location of the neutral axis, the following results (14-16) can be finally obtained for the normal force and bending moment due to compressive stresses in concrete:

$$N_b = 2 \cdot R^2 \cdot R_b \cdot N^*_b(\beta, \xi); \; M_b = 2 \cdot R^3 \cdot R_b \cdot M^*_b(\beta, \xi), \tag{14}$$

$$N^*_b(\beta, \xi) = \begin{cases} Y_{Nb}(\beta, \xi, -1, \rightleftarrows 1) - Y_{Nb}(\beta, \xi, -\gamma, \rightleftarrows \gamma); & if \quad \xi \geq 2 \\ Y_{Nb}(\beta, \xi, 1 - \xi, \rightleftarrows 1) - Y_{Nb}(\beta, \xi, -\gamma, \rightleftarrows \gamma); & if \quad 1 + \gamma \leq \xi < 2 \\ Y_{Nb}(\beta, \xi, 1 - \xi, \rightleftarrows 1) - Y_{Nb}(\beta, \xi, 1 - \xi, \rightleftarrows \gamma); & if \quad 1 - \gamma \leq \xi < 1 + \gamma \\ Y_{Nb}(\beta, \xi, 1 - \xi, \rightleftarrows 1); \quad if \quad \rightleftarrows & 0 < \xi < 1 - \gamma \end{cases}, \tag{15}$$

$$M^*_b(\beta, \xi) = \begin{cases} M_{Nb}(\beta, \xi, -1, \rightleftarrows 1) - M_{Nb}(\beta, \xi, -\gamma, \rightleftarrows \gamma); & if \quad \xi \geq 2 \\ M_{Nb}(\beta, \xi, 1 - \xi, \rightleftarrows 1) - M_{Nb}(\beta, \xi, -\gamma, \rightleftarrows \gamma); & if \quad 1 + \gamma \leq \xi < 2 \\ M_{Nb}(\beta, \xi, 1 - \xi, \rightleftarrows 1) - M_{Nb}(\beta, \xi, 1 - \xi, \rightleftarrows \gamma); & if \quad 1 - \gamma \leq \xi < 1 + \gamma \\ M_{Nb}(\beta, \xi, 1 - \xi, \rightleftarrows 1); \quad if \quad \rightleftarrows & 0 < \xi < 1 - \gamma \end{cases}. \tag{16}$$

The representation of the internal moment and the normal force in this form allows calculating these parameters using a single method. A software module has been compiled by the ABC Algorithmic Language for calculating these parameters. Based on the hypothesis of flat sections for deformation of arbitrary reinforcement (17):

$$\varepsilon_{sj} = = \frac{\beta \cdot \varepsilon_R}{\xi} \cdot \left(\xi - 1 + \frac{r_{sj}}{R} \cdot sin\, \phi_{sj}\right). \tag{17}$$

Then, according to the accepted reinforcement deformation diagram, the following expression (18) can be obtained for the normal stress of an arbitrary reinforcement:

$$\sigma_{sj} = \begin{cases} E_{sj} \cdot \frac{\beta \cdot \varepsilon_R}{\xi} \cdot \left(\xi - 1 + \frac{r_{sj}}{R} \cdot \sin\phi_{sj}\right) ; \left| \ if \ \frac{\beta \cdot \varepsilon_R}{\xi} \cdot \left(\xi - 1 + \frac{r_{sj}}{R} \cdot \sin\phi_{sj}\right) \right| \leq \varepsilon_{sj,\,ax} \\ R_{sj} ; \quad if \ \frac{\beta \cdot \varepsilon_R}{\xi} \cdot \left(\xi - 1 + \frac{r_{sj}}{R} \cdot \sin\phi_{sj}\right) > \varepsilon_{sj,\,ax} \\ -R_{sj} ; \ if \ \frac{\beta \cdot \varepsilon_R}{\xi} \cdot \left(\xi - 1 + \frac{r_{sj}}{R} \cdot \sin\phi_{sj}\right) < -\varepsilon_{sj,\,ax} \end{cases} . \quad (18)$$

In these expressions E_{sj}, R_{sj}, r_{sj}, ϕ_{sj}, $\varepsilon_{sj,ax}$ respectively, the modulus of elasticity, the tensile-compressive strength, the distance from the centre of the cross-section of the rod to the centre of the section of the reinforcement, the central angle, the deformation of the yield of the reinforcement (Fig. 2). According to the stresses found in the reinforcing bars, according to the following dependencies, it is possible to calculate the normal force and bending moment coming to the share of the reinforcing bars (19):

$$N_s(\beta, \xi) = \sum_{j=1}^{k_s} \sigma_{sj} \cdot A_{sj} ; \ M_s(\beta, \xi) = \sum_{j=1}^{k_s} \sigma_{sj} \cdot A_{sj} \cdot r_{sj} \cdot \sin\phi_{sj}, \quad (19)$$

where k_s – the number of reinforcing bars.

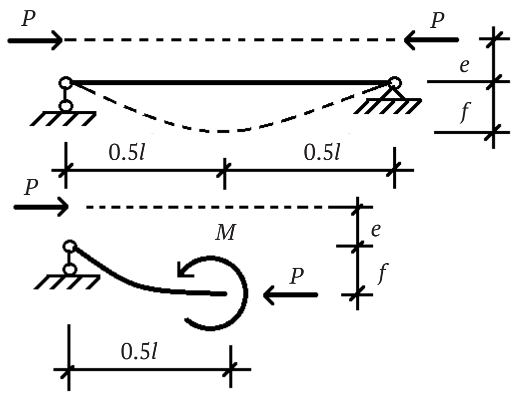

Figure 2. Design scheme of the compressed rod
Source: compiled by the authors

Composing the equilibrium equations for half of the rod can be written as (20, 21):

$$2 \cdot R^2 \cdot R_b \cdot N_b^*(\beta, \zeta) + N_s(\beta, \zeta) = P, \quad (20)$$

$$2 \cdot R^3 \cdot R_b \cdot M_b^*(\beta, \zeta) + M_s(\beta, \zeta) = P \cdot (e + f). \quad (21)$$

This nonlinear system of equations includes three unknown parameters β, ξ and f, but the maximum deflection of the rod can be expressed in terms of β and ξ. Approximating the deformed axis of the rod with a suitable curve, for example, for a pivotally supported rod: (22) (Kraus & Rimoli, 2023):

$$y(x) = f \cdot \sin\frac{\pi \cdot x}{l_0}, \quad (22)$$

where l_0 – estimated length of the compressed rod.

Then for the curvature of the most stressed middle section – $\chi = y''\left(\frac{l_0}{2}\right) = f \cdot \frac{\pi^2}{l_0^2}$, on the other hand – $\chi = \frac{\varepsilon_b}{x} = \frac{\beta \cdot \varepsilon_R}{\xi \cdot R}$. Equating these two expressions (23):

$$f = \rho_0 \cdot \frac{\beta}{\xi} ; \ \rho_0 = \frac{l_0^2 \cdot \varepsilon_R}{\pi^2 \cdot R}. \quad (23)$$

Thus, considering (23), the main resolving system of equations (20) and (21) are a nonlinear system with respect to the parameters β and ξ. Given (20) in (21), the following nonlinear equation can be obtained that relates the parameters β and ξ to each other:

$$\Phi(\beta, \zeta) = 2 \cdot R^3 \cdot R_b \cdot M_b^*(\beta, \zeta) + M_s^*(\beta, \zeta) - \left(e + \rho_* \cdot \frac{\beta}{\xi}\right) \cdot [2 \cdot R^2 \cdot R_b \cdot N_b^*(\beta, \zeta) + N_s^*(\beta, \zeta)] = 0. \quad (24)$$

From this equation, for a given value of one of the parameters, the second parameter is found as a solution to a nonlinear equation with one unknown. Since the limits of parameter change β are known in advance, its value is set and the corresponding value of parameter ξ is found. After that, by (23) it is possible to calculate the parameter f and by (21) it is possible to calculate the compressive force corresponding to the assumed value β. The remaining parameters at known values β and ξ are calculated easily. Thus, passing over the whole range of variation of parameter β, a connection is obtained between P and f, which allows building a load-deflection diagram. The maximum of this curve determines the bearing capacity of a compressed rod of annular section (Huang *et al.*, 2021). The described numerical calculation algorithm is easily programmed and the corresponding software module is compiled by the ABC Algorithmic Language. When constructing the computational model, no restrictions are made regarding the flexibility of the rod and the eccentricity of the compressive force application. The presence of such a software module allows for various numerical experiments. The results of some numerical experiments are given.

Example 1. The annular cross-section of the compressed element with the radius of the outer circle $R = 0.2$ m, the inner circle $r_{sj} = 0.1575$ m uniformly reinforced with $k_s = 12$ reinforcement of a periodic profile of a class *A* 400 with a diameter ∅ 25 of the cross-sectional area of each $A_{sj} = 4.9087 \cdot 10^{-4}$ m², $E_{sj} = 2 \cdot 10^5$ *MPa*, $R_{sj} = 350$ *MPa*, concrete *B* 25 with a compressive strength limit $R_b = 14.5$ *MPa*. To investigate the effect of the flexibility of the compressed element on the bearing capacity of a pivotally supported rod with conditionally central compression with eccentricity, $e = 1$ cm calculations were performed at the following lengths $l_0 = 3$ m, $l_0 = 4$ m, $l_0 = 6$ m, $l_0 = 9$ m, $l_0 = 12$ m and $l_0 = 15$ m. Based on the results of the performed calculation, Figure 3 shows the "load-deflection" graphs.

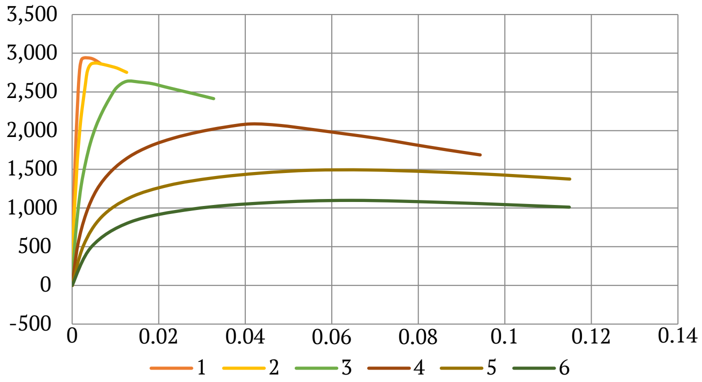

Figure 3. Load-deflection graphs for a conventionally centrally compressed rod at various lengths
Source: compiled by the authors

As can be seen in all graphs, descending branches are implemented, with an increase in the length of the rod, the bearing capacity decreases. The parameters of the stress-strain state at the moment of loss of bearing capacity are given in Table 1.

Table 1. Parameters of the stress-strain state at the moment of loss of bearing capacity

l_0; m	β	ξ	f; m	P; kN	$\frac{P_{cr}}{P}$
3	1.2	3.1708	0.0035	2,940.44	16.72
4	1.1	3.2621	0.0055	2,873.17	9.63
6	1.0	2.8936	0.0126	2,637.08	4.66
9	1.0	1.9892	0.0413	2,086.34	2.62
12	0.8	1.7052	0.0684	1,491.75	2.06
15	0.5	1.7658	0.0646	1,098.55	1.79

Source: compiled by the authors

Additionally, for small values of flexibility, the bearing capacity is mainly determined from the condition of strength or rigidity, and in flexible elements from the condition of stability (Fan *et al.*, 2023). For example, for the considered rods with a length of l_0 = 3 m, l_0 = 4 m and l_0 = 6 m at the moment of exhaustion of the bearing capacity, the reinforcing rods with numbers 3, 4 and 5 reach the yield strength, and the remaining rods work within the limits of elasticity. For a rod with a length l_0 = 9 m – only the reinforcement with the number 4 reaches the yield strength. And in rods with a length l_0 = 12 m and l_0 = 15 m at the moment of exhaustion of the bearing capacity, all reinforcing rods work within the limits of elasticity.

Example 2. To clarify the effect of the eccentricity of the compressive force on the bearing capacity, the rods considered in the first example were calculated for different values of eccentricity. Table 2 provides the parameters that correspond to the exhaustion of the bearing capacity.

Table 2. Parameters corresponding to the exhaustion of the bearing capacity

l_0 = 3 m				
e, m	β	ξ	f, m	P, kN
0.01	1.2	3.1708	0.0035	2,940.44
0.02	1.3	2.3902	0.005	2,691.49
0.05	1.5	1.7929	0.0076	2,186.31
0.1	1.6	1.4557	0.01	1,632.69
0.15	1.7	1.2996	0.0119	1,295.91
l_0 = 4 m				
e, m	β	ξ	f, m	P, kN
0.01	1.1	3.2621	0.0055	2,873.17
0.02	1.2	2.3786	0.0082	2,612.37
0.05	1.4	1.7654	0.0129	2,108.77
0.1	1.6	1.4224	0.0182	1,567.4
0.15	1.7	1.2796	0.0215	1,247.39

Table 2. Contnued

		l_0 = 6 m		
e, m	*β*	*ξ*	*f*, m	*P*, kN
0.01	1.0	2.8936	0.0126	2,637.08
0.02	1.2	2.0727	0.0211	2,372.39
0.05	1.4	1.622	0.0315	1,889.16
0.1	1.6	1.3389	0.0436	1,394.06
0.15	1.7	1.2304	0.0504	1,118.29
		l_0 = 9 m		
e, m	*β*	*ξ*	*f*, m	*P*, kN
0.01	1.0	1.9892	0.0413	2,086.34
0.02	1.0	1.8243	0.045	1,898.13
0.05	1.1	1.4592	0.0619	1,479.93
0.1	1.2	1.2317	0.08	1,099.59
0.15	1.3	1.1433	0.0933	893.31
		l_0 = 12 m		
e, m	*β*	*ξ*	*f*, m	*P*, kN
0.01	0.8	1.7052	0.0684	1,491.75
0.02	0.8	1.5775	0.074	1,345.14
0.05	1.0	1.2525	0.1165	1,051.42
0.1	1.2	1.1188	0.1565	832.1
0.15	1.2	1.0714	0.1634	702.88
		l_0 = 15 m		
e, m	*β*	*ξ*	*f*, m	*P*, kN
0.01	0.5	1.7658	0.0646	1,098.55
0.02	0.5	1.6174	0.0705	984.34
0.05	0.7	1.1952	0.1335	752.1
0.1	1.1	1.046	0.2397	623.47
0.15	1.2	1.0151	0.2695	543.32

Source: compiled by the authors

Similarly, for a rod with a length l_0 = 4 m in Figure 4 shows the "load-deflection" graphs. From these graphs, it is seen that for small eccentricities, the bearing capacity is determined from the stability condition, and for large values of eccentricity from the strength or stiffness condition. With large values of eccentricity, descending branches are not implemented in load-deflection diagrams.

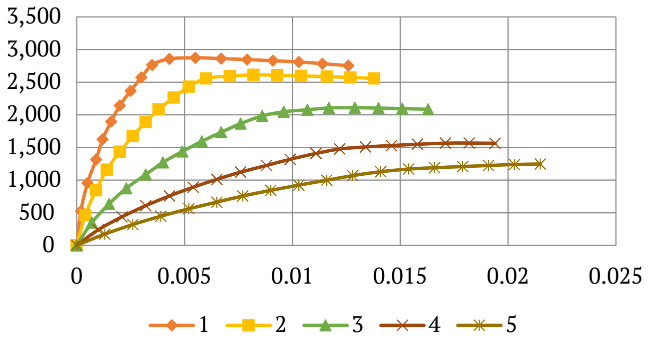

Figure 4. Load-deflection diagrams for an element with a length l_0 = 4 m at different values of eccentricity
Source: compiled by the authors

Example 3. The influence of the percentage of reinforcement on the bearing capacity of compressed reinforced concrete elements of the annular section is studied here. For this purpose, an element with a length l_0 = 4 m with a conditionally central compression with an eccentricity *e* = 1 cm was additionally calculated with the following reinforcements 12 Ø 22, 12 Ø 20, 12 Ø 18, and 12 Ø 16.

The values corresponding to the point of bearing capacity exhaustion can be found in Table 3.

Table 3. Values of the parameters corresponding to the moment of exhaustion of the bearing capacity

Reinforced	β	ξ	f, m	P, kN
12 ∅ 25	1	2.4845	0.0205	2,459.24
12 ∅ 22	1.1	3.2621	0.0055	2,873.17
12 ∅ 20	1.1	3.1678	0.0056	2,464.73
12 ∅ 18	1.1	3.0914	0.0058	2,221.28
12 ∅ 16	1.1	3.004	0.0059	2,000.49

Source: compiled by the authors

The percentage of reinforcement also strongly affects the bearing capacity.

"Load-deflection" plots for the considered reinforcement options are plotted in Figure 5.

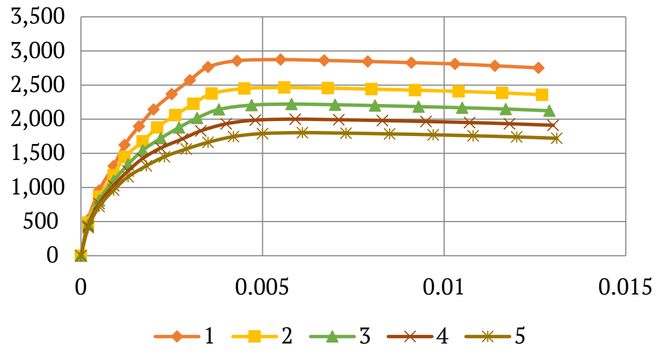

Figure 5. Load-deflection graphs for an element with a length l_0 = 4 m with conditionally central compression with eccentricity e = 1 cm for various reinforcement options

Source: compiled by the authors

These graphs show the significant influence of the percentage of reinforcement on the behaviour of compressed reinforced concrete elements of the annular section. Changing the reinforcement level significantly affects the bearing capacity. A higher percentage of reinforcement can significantly increase the load-bearing capacity, making the structure more reliable and able to withstand higher loads. These results are of great importance in the development of engineering solutions and optimisation of structures with annular elements. By controlling and regulating the percentage of reinforcement, engineers can achieve the desired characteristics in the behaviour of the structure, which contributes to the safety and efficiency of engineering projects.

Thus, the analysis of the influence of the percentage of reinforcement on the behaviour of compressed reinforced concrete elements of the annular section emphasises the importance of optimal design and control during construction to achieve optimal results in constructive work.

DISCUSSION

Annular cross-section elements are widely used in infrastructure projects such as bridges, stadiums, towers, and even ordinary buildings. They provide not only a supporting function, but can also be important elements of design and architectural value. Insufficient understanding and incorrect design of such elements can lead to accidents and serious consequences.

The stress-strain state of compressed reinforced concrete elements of the annular section is influenced by several factors, including the geometry of the element, the class of concrete and reinforcement used, and the loads acting on the element. Engineers can use knowledge of the stress-strain state to design the elements of the annular section accurately. This involves optimizing the element's geometry, selecting materials, and placing the reinforcement correctly to ensure maximum load-bearing capacity and safety. Failure to accurately estimate the stress-strain state can result in excessive material consumption or even the destruction of the element.

Reinforcement plays a key role in increasing the load-bearing capacity of compressed reinforced concrete elements. The correct placement and number of fittings help to control the stress-strain state and prevent the destruction of the element during compression. Various methods are used to analyse the stress-strain state and assess the bearing capacity of annular elements. This includes numerical modelling, laboratory mock-up tests and analytical calculations. Modern engineering programmes and computer simulations help to more accurately assess the behaviour of such elements.

The application of this research is seen in the design and construction of many structures, including bridges, water towers, and in industrial structures. For example, when designing high towers with annular elements, it is necessary to consider the stress-strain state to ensure their stability and safety. Modern research in this field is aimed at developing new materials, methods of analysis and design, and at finding sustainable and innovative solutions to improve the bearing capacity and durability of compressed reinforced concrete elements. Problems may include insufficient reinforcement, incorrect design of the geometry of elements, changes in operating conditions over time, and many other factors. Solutions often require engineering analysis and design modification. Discussion of all these aspects will help to better understand the importance and relevance of this topic in engineering practice and emphasise the need for continuous development in this area to ensure the safety and sustainability of infrastructure facilities.

According to the results of research by X. Wang *et al.* (2015), the behaviour of short round tubular reinforced concrete columns subjected to off-centre compression is an important research topic in the field of structural mechanics and engineering design. This type of structure is commonly found in a variety of engineering applications, such as bridge supports, poles, wind turbine supports, and many others. Off-centre compression means that the load axis does not coincide with the geometric centre of the column. This can occur due to various factors, including the eccentricity of the load application or heterogeneity within the structure. Due to this uneven load distribution, additional stresses and deformations occur in the column, which can significantly affect its bearing capacity and stability. Results obtained by the researchers are consistent with the findings of this study. Research in this field is focused on the development of numerical models and analytical methods for the analysis of such columns. These scientific studies allow engineers to more accurately predict the behaviour of columns and optimise their design considering the impact of eccentric loads. The essential aspects of the analysis are the assessment of normal and transverse stresses, stability testing and the development of safety measures to prevent possible emergencies. Understanding the characteristics of short round tubular reinforced concrete columns under eccentric compression is critical to ensuring the reliability and safety of engineering structures.

S. Indriūnas *et al.* (2023) have found that the behavioural analysis of a mast with a combined system, including pre-stressed columns and a core of centrifuged concrete of circular cross-section, is an interesting area of research in engineering construction. This approach combines two key elements to ensure high load-bearing capacity and stability of the mast. Pre-stressed columns are an effective way to strengthen the structure and increase its load-bearing capacity. They create compression stresses that compensate for part of the load and increase resistance to bending moments. The core of centrifuged concrete of circular cross-section, in turn, provides additional structural strength and stability. Behavioural analysis of such a mast includes the study of many aspects, including stress distribution in pre-stressed columns, deformations and dynamic behaviour under various loads and operating conditions (Chovnyuk *et al.*, 2022). In addition, it is important to consider the interaction between the various elements of the system, such as columns and core, to ensure their coordinated operation. These data correspond to the statements set out in the previous section and in earlier researches in this field. For example, Q.-L. Wang *et al.* (2008) analysed of load-deformation curves of the concentrically compressed concrete and got similar conclusions. From which it can be inferred that research in this field can contribute to optimising the design of such masts, providing high load-bearing capacity and durability with minimal use of materials and economical design. This is especially relevant in the context of the construction of tall buildings and towers, where security and stability play a crucial role.

R. Kliukas *et al.* (2020) determined that the mechanical properties of centrifuged concrete play an important role in reinforced concrete structures. Centrifuged concrete is a specific type of concrete that is subjected to high centrifugal forces during production. This allows achieving a higher density and strength of the material compared to conventional concrete, which makes it an attractive choice for various engineering applications. One of the key mechanical properties of centrifuged concrete is its compressive strength. This parameter determines the ability of the material to withstand the load directed along its axis. The high compressive strength makes centrifuged concrete ideal for use in compressible elements such as pillars, columns and foundations. The paper confirms the results of this study. The mechanical characteristics of centrifuged concrete cover parameters such as tensile strength, modulus of elasticity, and resistance to fracture during bending, among other factors. The understanding of these properties is an important aspect in the process of designing and analysing reinforced concrete structures. This contributes to ensuring the safety and reliability of structures, and also allows optimising their design based on the requirements for loads and durability.

P. Feng *et al.* (2015) determined that the study of the mechanical behaviour of a square steel pipe with a concrete core subjected to axial compression is an interesting area in the engineering and scientific community. Such composite structures combine the advantages of various materials and can be used in various engineering applications, such as the construction of bridges, towers, and other structures where a combination of high strength and stability is required. The results of the author's research can reveal possible problems and limitations in the use of this type of composite materials in various engineering projects, which contributes to more efficient and safe design and construction. Thus, the analysis of the mechanical behaviour of such structures under axial compression is an important stage in the research of engineers and scientists.

Researchers M. Kosior-Kazberuk *et al.* (2022) have shown that the approximation model of the method of calculated resistance of reinforced concrete to bent elements is an important engineering tool that allows engineers to evaluate the bearing capacity of reinforced concrete structures under the influence of bending moments. This method uses various simplifications and approximations to predict the behaviour of reinforced concrete elements during bending and can be useful in the design and analysis of building structures. The approximation model includes parameters such as concrete strength, reinforcement strength, and geometric characteristics of the element. It can be used to determine critical parameters, such as the maximum bending moment that the structure can withstand without breaking. These results confirm the above study, as this method allows engineers to carry out preliminary calculations and analysis of structures more quickly and efficiently, which reduces risks and ensures safety in construction. However, it is important to remember that the results obtained using approximation models should always be supplemented with more accurate and detailed studies and experiments to ensure the reliability and accuracy of calculations.

A. Pavlikov *et al.* (2020) examined the strength characteristics of compressed reinforced concrete elements with a circular cross-section and uniform reinforcement under the influence of axial load and bending moment. They proposed a simple method of analysis for engineering practice that reduces the basic formulas of nonlinear analysis to the formulas of the resistance of materials. The results of their study are consistent with the data obtained, and the use of nonlinear diagrams of deformation of materials allows for a more accurate description of the real behaviour of reinforced concrete elements. Developing simplified engineering techniques based on nonlinear analysis is an important direction for improving the design efficiency of such structures. However, it is important to keep in mind the limitations of such simplified models and the need to verify the results using more detailed numerical and experimental studies.

Overall, the mentioned studies represent a valuable contribution to advancing the methods for analysing and designing non-centrally compressed reinforced concrete elements with a circular cross-section.

CONCLUSIONS

Elements, such as pillars, columns, or supports, are frequently subjected to compression loads, and understanding their behaviour during deformation is crucial for guaranteeing the safety and durability of building structures. A robust numerical method has been developed in this study to explore the stress-strain state and load-bearing capacity of compressed reinforced concrete elements with an annular cross-section. This technique enables the examination of the behaviour of reinforced concrete elements with an annular section using a unified approach, without imposing limitations on flexibility, force application eccentricity, or the percentage of reinforcement. The results of numerical experiments affirm the necessity of employing a nonlinear deformation model based on real material deformation diagrams for a reliable analysis of the stress-strain state and bearing capacity of compressed reinforced concrete elements. This makes the method appealing for use in constructing bridges, buildings, and other infrastructure facilities. However, it is essential to consider various factors during the design process, such as the element's geometry, the quality of the concrete and reinforcement, and environmental conditions like exposure to aggressive media and temperature changes, as neglecting these parameters may result in the element's destruction or loss of bearing capacity.

It is worth noting that ongoing research and development in the field of building materials and technologies are continuously enhancing the bearing capacity of compressed reinforced concrete annular section elements and improving safety in construction. In conclusion, understanding the stress-strain state and bearing capacity of such elements remains a pivotal aspect in the field of engineering practice and construction, necessitating further research and development. For further exploration of this topic, a more comprehensive investigation into the impact of long-term operation, dynamic loads, and alternative building materials on the bearing capacity of compressed concrete elements with an annular cross-section is essential.

ACKNOWLEDGEMENTS

None.

CONFLICT OF INTEREST

None.

REFERENCES

[1] Azizov, T. (2021). Determination of displacements of a reinforced concrete element in a normal crack. *Sciences of Europe*, 62, 34-38. doi: 10.24412/3162-2364-2021-62-1-34-38.

[2] Baghirzade, N. (2022). Calculation of wooden elements based on the nonlinear deformation model. In V. Onyshchenko, G. Mammadova, S. Sivitska, & A. Gasimov (Eds.), *Proceedings of the 3rd international conference on building innovations* (pp. 43-48). Cham: Springer. doi: 10.1007/978-3-030-85043-2_5.

[3] Chovnyuk, Y., Diachenko, L., Ivanov, Y., Dichek, N., & Orel, O. (2022). Optimisation of dynamic loads of rope systems of lifting mechanisms of bridge cranes during cargo handling. *Scientific Herald of Uzhhorod University. Series "Physics"*, 51, 59-73. doi: 10.54919/2415-8038.2022.51.59-73.

[4] Fan, Y., Luo, C., Zhou, Y., Yang, L., Li, X., & Liao, J. (2023). Evolution law of concrete interface stress of rigid-frame arch under construction and its impact on ultimate load-bearing capacity. *Sensors*, 23(15), 6868. doi: 10.3390/s23156868.

[5] Feng, P., Cheng, S., Bai, Y., & Ye, L. (2015). Mechanical behavior of concrete-filled square steel tube with FRP-confined concrete core subjected to axial compression. *Composite Structures*, 123, 312-324. doi: 10.1016/j.compstruct.2014.12.053.

[6] Huang, T., Zhang, D., Yang, J., Lin, Y., & Huang, Y. (2021). Compressive bearing capacity and failure mechanism of CFRP-aluminum laminate column with single-channel cross section. *Science and Engineering of Composite Materials*, 28(1), 48-63. doi: 10.1515/secm-2021-0004.

[7] Indriūnas, S., Kliukas, R., & Juozapaitis, A. (2023). Behavioral analysis of a mast with a combined prestressed stayed columns system and core of a spun concrete circular cross-section. *Buildings*, 13(9), 2175. doi: 10.3390/buildings13092175.

[8] Janahmadov, A., Volchenko, A., Nasirova, M., Volchenko, N., Vitvitskiy, V., & Fedotov, E. (2021). Research on hydrodynamic parameters of wet friction pairs of disc-shoe brake of vehicle. *Herald of the Azerbaijan Engineering Academy*, 13(1), 32-42. doi: 10.52171/2076-0515_2021_13_01_32_42.

[9] Jian, B., Weng, X., Lin, Y., & Li, P. (2023). Study on the seismic performance of prestressed steel-concrete vierendeel truss transfer story frame. *Structures*, 57, 105123. doi: 10.1016/j.istruc.2023.105123.

[10] Kliukas, R., Lukoševičienė, O., Jaras, A., & Jonaitis, B. (2020). The mechanical properties of centrifuged concrete in reinforced concrete structures. *Applied Sciences*, 10(10), 3570. doi: 10.3390/app10103570.

[11] Kosior-Kazberuk, M., Kochkarev, D., Azizov, T., & Galinska, T. (2022). Approximation model of the method of design resistance of reinforced concrete for bending elements. In V. Onyshchenko, G. Mammadova, S. Sivitska, & A. Gasimov (Eds.), *Proceedings of the 3rd international conference on building innovations* (pp. 245-254). Cham: Springer. doi: 10.1007/978-3-030-85043-2_23.

[12] Kraus, J.A., & Rimoli, J.J. (2023). An elastica theory for compressible imperfect beams with application to mechanical metamaterials. *Mechanics Research Communications*, 131, 104147. doi: 10.1016/j.mechrescom.2023.104147.

[13] Pavlikov, A., Kochkarev, D., & Harkava, O. (2020). Analysis of eccentrically loaded members of circular cross section by nonlinear deformation model. In V. Onyshchenko, G. Mammadova, S. Sivitska, & A. Gasimov (Eds.), *Proceedings of the 2nd international conference on building innovations* (pp. 171-181). Cham: Springer. doi: 10.1007/978-3-030-42939-3_19.

[14] Ruiz, G., De La Rosa, Á., Poveda, E., Zanon, R., Schäfer, M., & Wolf, S. (2023). Compressive behavior of steel-fiber reinforced concrete in Annex L of new Eurocode 2. *Hormigón y Acero*, 74(299-300), 187-198. doi: 10.33586/hya.2022.3092.

[15] Sadeghian, F., Haddad, A., Jahandari, S., Rasekh, H., & Ozbakkaloglu, T. (2021). Effects of electrokinetic phenomena on the load-bearing capacity of different steel and concrete piles: A small-scale experimental study. *Canadian Geotechnical Journal*, 58(5), 741-746. doi: 10.1139/cgj-2019-0650.

[16] Seyfullaev, Kh., & Garaev, A. (2018). Application of a nonlinear deformation model to the calculation of bendable reinforced concrete elements. *Sciences of Europe*, 33, 51-60. Retrieved from https://cyberleninka.ru/article/n/prilozhenie-nelineynoy-deformatsionnoy-modeli-k-raschetuizgibaemyh-zhelezobetonnyh-elementov.

[17] Spirande, K., Shemet, R., Iakymenko, M., & Shemet, K. (2023). Analysis of influence of design parameters on the operation of composite steel and concrete modified I-beams. *AIP Conference Proceedings*, 2684, 030041. doi: 10.1063/5.0120406.

[18] Wang, Q.-L., Gao, G.-R., & Ren, Q.-X. (2008). Analysis on load-deformation curves of the concentrically compressed concrete filled circular CFRP-steel tubular stub columns. *Key Engineering Materials*, 400-402, 507-512. doi: 10.4028/www.scientific.net/KEM.400-402.507.

[19] Wang, X., Liu, J., & Zhang, S. (2015). Behavior of short circular tubed-reinforced-concrete columns subjected to eccentric compression. *Engineering Structures*, 105, 77-86. doi: 10.1016/j.engstruct.2015.10.001.

[20] Zhang, Q., Wei, Z.-Y., Gu, X.-L., Yang, Q.-C., Li, S.-Y., & Zhao, Y.-S. (2022). Confinement behavior and stress-strain response of square concrete columns strengthened with carbon textile reinforced concrete (CTRC) composites. *Engineering Structures*, 266, 114592. doi: 10.1016/j.engstruct.2022.114592.

Мухліс Ахмед огли Гаджиєв
Доктор технічних наук, професор
Азербайджанський архітектурно-будівельний університет
AZ1073, вул. Айни Султанової, 11, м. Баку, Азербайджан
https://orcid.org/0000-0001-6782-0941
Масіл Махір огли Даміров
Аспірант, старший викладач
Азербайджанський архітектурно-будівельний університет
AZ1073, вул. Айни Султанової, 11, м. Баку, Азербайджан
https://orcid.org/0000-0002-3779-4305

Напружено-деформований стан і несуча здатність стиснутих залізобетонних елементів кільцевого перерізу

Анотація. Дослідження напружено-деформованого стану та несучої здатності стиснутих залізобетонних елементів кільцевого перерізу є актуальним з огляду на постійно зростаючу потребу в удосконаленні та оптимізації об'єктів інфраструктури, таких як мости та опори, для забезпечення їх надійності та безпеки. Метою даної роботи є дослідження та аналіз напружено-деформованого стану стиснутих залізобетонних елементів кільцевого перерізу для визначення їх несучої здатності та підвищення ефективності проектування і будівництва об'єктів інфраструктури. Слід виділити аналітичний метод, класифікацію, функціональний, статистичний, синтез та інші методи, що були використані в роботі. Стиснуті залізобетонні елементи кільцевого перерізу широко застосовуються в різних галузях машинобудування та будівництва. Однак з 90-х років 20 ст. спостерігається помітний розвиток нелінійної теорії деформування для розрахунку залізобетонних конструкцій, що ускладнює аналіз через специфічні особливості круглих і кільцевих перерізів і призводить до відсутності простих аналітичних методів. У цій роботі представлено ефективну чисельну методику дослідження напружено-деформованого стану та несучої здатності нецентрально стиснутих елементів, яка використовує лінійну дробову залежність для бетону при стисненні, затверджену в Єврокоді, та симетричну дволінійну діаграму деформування арматури. Важливо підкреслити, що ця методика дозволяє уникнути необхідності класифікувати елементи на короткі та довгі, а також окремо враховувати малі та великі ексцентриситети, оскільки ці аспекти автоматично враховуються в розрахунках. Підтвердження ефективності цієї методики отримано за допомогою результатів чисельних експериментів. Практичне значення роботи полягає в розробці більш точних і надійних методів розрахунку та проектування стиснутих залізобетонних елементів кільцевого перерізу, що сприяє підвищенню безпеки і довговічності об'єктів інфраструктури та зменшенню ризику їх руйнування

Ключові слова: арматура; ексцентриситет; діаграма «навантаження-прогин»; інфраструктурні об'єкти; чисельна методика